

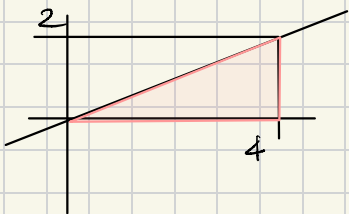
## 5.4 Changing the order of integration

We have already seen that it may be possible to integrate over a region in two ways:  $x$  first,  $y$  second; or  $y$  first,  $x$  second. Sometimes we can't do the integration one way round, but we can do it the other way.

Example: 
$$\int_0^2 \int_{2y}^4 e^{x^2} dx dy$$

Problem: I don't know how to do

$$\int_a^b e^{x^2} dx$$



Solution: Write the integral the

other way, to get

$$\int_0^4 \int_0^{\frac{1}{2}x} e^{x^2} dy dx$$

$$= \int_0^4 \left[ y e^{x^2} \right]_0^{\frac{x}{2}} dx$$

$$= \int_0^4 \frac{x}{2} e^{x^2} dx = \left[ \frac{e^{x^2}}{4} \right]_0^4$$

$$= \frac{e^{16} - 1}{4}$$

## Mean value inequality and equality for integrals

If  $m \leq f(x,y) \leq M$  for all  $(x,y)$  in  $D$  then

$$m A(D) \leq \iint_D f \, dA \leq M A(D)$$

where  $A(D)$  is the area of  $D$ .

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$$\frac{1}{\sqrt{3}} \leq \iint_{[0,1] \times [0,1]} \frac{1}{\sqrt{1+x^6+y^6}} \, dA \leq 1$$

Theorem 5. If  $f$  is continuous then for some  $(x_0, y_0)$  in  $D$  we have

$$\iint_D f(x,y) \, dA = f(x_0, y_0) A(D)$$

